MANAV RACHNA INTERNATIONAL INSTITUTE OF RESEARCH & STUDIES
(Deemed to be University under section 3 of the UGC Act 1956)

Ph.D ADMISSION TEST (MR-PAT)

Ph.D. in Mathematics

Module 1: Analysis

1.1 Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Module 2: Linear Algebra

2.1 Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms

Module 3: Complex Analysis

3.1 Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Module 4: Algebra

4.1 Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory. Topology: basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

Module 5: Differential Equations

5.1 Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function. Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Module 6: Numerical Analysis

6.1 Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Module 7: Calculus of Variations

7.1 Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Module 8: Linear Integral Equations

8.1 Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Suggested Readings:

1. P.M. Cohn, Basic Algebra: Groups, Rings and Fields, Springer, 2005.
2. D.S. Dummit and R.M. Foote, Abstract Algebra, Third Edition, Wiley, 2011.
3. N. Jacobson, Basic Algebra, Volumes I & II, Second Edition, Dover Publications, 2009.
4. T.W. Hungerford, Algebra, Springer-Verlag, 1981.
5. T.W. Gamelin, Complex Analysis, Springer, 2001.
6. E. Hewitt and K. Stromberg, Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable, Springer, Berlin, 1975.
7. E.A. Coddington, An Introduction to Ordinary Differential Equations, Dover Publications, 2012.
8. S.L. Ross, Differential Equations, Second Edition, John Wiley & Sons, India, 2007.
9. I.N. Sneddon, Elements of Partial Differential Equations, Dover Publications, 2006.
10. J.L. Kelley, General Topology, Dover Publications, 2017.